UNIVERSITÉ TOULOUSE 1 CAPITOLE

Année Universitaire 2012-2013
SEMESTRE 2
SESSION 2

MASTER 1 in ECONOMICS
MASTER 1 ECONOMIE ET STATISTIQUE + MAGISTERE

Times series / code : M1S212

Lundi 1er juillet 2013

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➥ durée conseillée pour traiter **ce** sujet : 1 heure
➥ **ATTENTION** : le nom de la matière et son code doivent être **IMPERATIVEMENT** recopiés sur la copie d'examen

Question A:

Compute the ACF(1), ACF(2), PACF(1), and PACF(2) of an ARMA(1,1).

Question B:

Consider the model

$$y_t = \omega + \beta_1 x_{t-1} + \beta_2 x_{t-2} + u_t$$
$$x_t = \psi + \rho x_{t-1} + v_t$$

where

$$(u_t, v_t)^\top \quad \text{i.i.d.} \quad \sim \quad \mathcal{N}\left(0, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}\right).$$

1. You want to estimate the vector $\beta = (\beta_1, \beta_2)'$.
   (a) Which method do you propose?
   (b) What are the properties in finite sample of the estimator? Be precise, in particular about the parameters that may play a role in these properties.
   (c) What are the asymptotic properties of the estimator?
2. Now, you consider the three-step ahead forecast regression, i.e., the regression of $y_{t+1} + y_{t+2} + y_{t+3}$ on the constant, x_t, and x_{t-1}, with $t = 1, ..., T-2$.
   (a) What is the population value of the constant and slope parameters?
   (b) How do you estimate the asymptotic variance of the OLS estimator? Give the details.
3. Characterize the ARMA dynamics of y_t. You have to find the order of the ARMA and the parameters.
4. Compute and compare the forecast of y_{t+1} given x_t (and its past values) and the forecast of y_{t+1} given y_t (and its past values). (You should include the constant.)
5. Compute the forecast of y_{t+h} given x_t, its past values, and a constant, at any horizon h.