

MASTER 1 in ECONOMICS  
MASTER 1 ECONOMIE ET STATISTIQUE

Stochastic Process / code : M1S27

Lundi 1<sup>er</sup> juillet 2013

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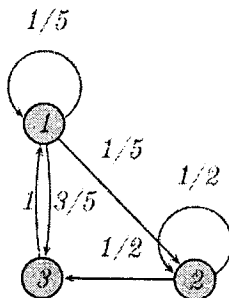
O. FAUGERAS

↳ durée conseillée pour traiter ce sujet : 1 heure

↳ ATTENTION : le nom de la matière et son code doivent être IMPERATIVEMENT recopiés sur la copie d'examen

*Closed book. Calculator authorised. Exercises are independent.  
It is advised to provide careful reasoning and justifications in your answers. It will be taken a great care of them in the notation.*

**Exercise 1** Let the homogeneous Markov chain  $(X_n)_{n \in \mathbb{N}}$  be described by the following graph :



The chain starts with the following initial distribution  $\alpha_0 = (0.2 \ 0.4 \ 0.4)$ . Compute

1.  $P(X_3 = 1 | X_1 = 1)$ ,
2.  $P(X_2 = 2)$ ,
3.  $P(X_0 = 1, X_2 = 1)$ ,
4.  $P(X_{100} = 1 | X_{99} = 3, X_{54} = 2)$ ,
5. What is the invariant distribution? What happens to the chain if the initial distribution is equal to the invariant distribution?
6. What is the average long run proportion of time spent in state 2?

**Exercise 2** Conditional expectation

Let  $X, Y$  be two  $\mathcal{N}(0, 1)$  independent random variables. Set  $Z = X + Y$ . Compute  $E(Z|X)$  and  $E(X|Z)$ .