

MASTER 1 in ECONOMICS  
MASTER 1 ECONOMIE ET STATISTIQUE

**Numerical optimization / code : M1S28**

Lundi 1<sup>er</sup> juillet 2013

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I. SCHINDLER

↳ durée conseillée pour traiter ce sujet : 1 heure

↳ ATTENTION : le nom de la matière et son code doivent être IMPERATIVEMENT recopiés sur la copie d'examen

**Exercice 1.** Let  $A \in M_n(\mathbb{R})$  be symmetric. Show that the  $A$  is positive definite if and only if it's eigenvalues are all positive.

**Exercice 2.** Let  $f(x) = (x - 2)^2$ .

- Explain why the theorem for Newton's method does not apply to this function.
- Show that Newton's formula converges to the zero which we will call  $a$ .
- Give an interval  $I$  such that for any  $x_0 \in I$ , Newton's formula converges to  $a$ .
- If  $x_0 = -1$ , give a formula to estimate  $|x_p - a|$  where  $x_p$  is the  $p$ 'th iteration of Newton's formula.
- In what way does the theorem for Newton's method fail ?

**Exercice 3.** Let

$$A = \begin{pmatrix} 2 & -2 \\ -2 & 4 \end{pmatrix} \text{ and } b = \begin{pmatrix} 4 \\ 8 \end{pmatrix}.$$

1. Compute the vectors  $u^1$  obtained by the iterative methods of Jacobi, Gauss-Seidel, and relaxation with  $\omega = 1,5$  for solving  $Au = b$  with  $u^0 = [1 \ 1]^T$ .
2. Show that these methods converge for  $A$ .

**Exercice 4.** Resoudre au sens des moindres carrés le système :

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 1 \\ x_1 + 3x_2 &= 1 \end{aligned}$$