Année Universitaire 2012-2013
SEMESTRE 2
SESSION 2

UNIVERSITÉ TOULOUSE 1 CAPITOLE

MASTER 1 in ECONOMICS
MASTER 1 ECONOMIE ET DROIT
MASTER 1 ECONOMIE ET STATISTIQUE

Industrial Organization / code : M1S21

Lundi 1er juillet 2013

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→ durée conseillée pour traiter **ce** sujet : 1 heure
→ **ATTENTION** : le nom de la matière et son code doivent être **IMPERATIVEMENT** recopiés sur la copie d'examen

Exercise 1 (14 points):

Consider a street represented by the interval $[0,1]$. There is a mass, normalized to 1, of consumers uniformly distributed along this street. Firm A, which is located at $x = a \in \left[0, \frac{1}{2}\right]$, and firm B, which is located at $x = 1$, sell a good for which each consumer has a unit demand. When a consumer buys one unit of the good at price p_i from firm $i = A, B$ his/her net utility is given by $\bar{s} - p_i - td_i^2$ where $\bar{s}$ is the gross utility from consuming one unit of the good, $t > 0$ is a transportation cost parameter and d_i is the distance between the consumer and firm i (note that transportation costs are quadratic). Assume that both firms produce at the same marginal cost $c > 0$.

Firms A and B play the following two-stage game:
Stage 1: Firm A chooses its location $a \in \left[0, \frac{1}{2}\right]$.
Stage 2: Firms A and B set their prices p_A and p_B simultaneously.

We assume that the parameters of the model are such that the whole market is covered in the equilibrium of the price-competition subgame (stage 2) for any $a \in \left[0, \frac{1}{2}\right]$.

1. Let us start with stage 2.

1.1. (2 pt) For a given value $a \in \left[0, \frac{1}{2}\right]$, compute the location of the indifferent consumer as a function of p_A, p_B and a. Write down the demand functions $D_A(p_A, p_B, a)$ and $D_B(p_A, p_B, a)$ and the profit functions $\pi_A(p_A, p_B, a)$ and $\pi_B(p_A, p_B, a)$ of firms A and B respectively (you may restrict to the price pairs (p_A, p_B) for which both demands are strictly positive as this will be the case in equilibrium).

1.2. (4 pt) Compute the best-response function of each firm and then show that the equilibrium prices at stage 2 for a given value $a \in \left[0, \frac{1}{2}\right]$ are:

$$p_A^*(a) = c + t(1-a)\left(1 + \frac{a}{3}\right);\ p_B^*(a) = c + t(1-a)\left(1 - \frac{a}{3}\right)$$

2. Consider now stage 1 and denote $\Pi_A(a) \equiv \pi_A(p_A^*(a), p_B^*(a), a)$.

2.a. (5 pt) Write down the (total) derivative $\frac{d\Pi_A}{da}$ as the sum of three terms using the partial derivatives $\frac{\partial \pi_A}{\partial p_A}$, $\frac{\partial \pi_A}{\partial p_B}$ and $\frac{\partial \pi_A}{\partial a}$ and the derivatives $\frac{dp_A^*}{da}$ and $\frac{dp_B^*}{da}$. Determine the sign of each of these three terms (strictly positive, zero, or strictly negative). What are the two effects that an increase in a has on firm A's profit $\Pi_A(a)$?

2.b. (3 pt) Find firm A's optimal location a^*. Discuss.

Exercise 2 (6 points):

Consider 2 firms producing a homogeneous good and choosing output levels in each period of an infinitely repeated game. The inverse demand for this good is given by $p = 1 - Q$ where Q is the total output. The two firms are identical: they have the same discount factor δ and they both produce at marginal cost $c = 0$.

Consider the following trigger strategies. Each firm sets the output $q^c = \frac{1}{4}$ at the beginning of the game, and continues to do so unless the other firm deviates. If a deviation occurs, each firm sets the quantity q^* corresponding to the Nash equilibrium of the static Cournot game for all the remaining periods.

Find the condition on δ for collusion (using the described strategies) to be sustainable.