UNIVERSITÉ TOULOUSE 1 CAPITOLE

TSE
Toulouse School of Economics
École d'économie de Toulouse

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SESSION 1

MASTER 1

INDUSTRIAL ORGANIZATION
(durée 3h00)

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Mardi 7 mai 2013 ~ 08h30 - 11h30

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Calculators are not authorized - You may answer either in English or in French but you must use the same language for all questions.

Problem 1 (10 points)
Quality, advertising and agglomeration in the Hotelling model

1. *Baseline model*

Consider a street represented by the interval $[0, 1]$. There is a unit mass of consumers uniformly distributed along the street. Firm A, which is located at $x = 0$, and firm B, which is located at $x = 1$, sell a good for which consumers have a unit demand. The net utility of a consumer who buys one unit of the good at price p_i from firm $i = A, B$ is given by $\bar{s} - p_i - td_i$ where $\bar{s}$ is the gross utility from consuming one unit of the good, $t > 0$ is the unit transportation cost and d_i is the distance between the consumer and firm i. Assume that both firms have the same marginal cost $c > 0$ (and no fixed costs). Firms A and B set their prices p_A and p_B simultaneously.

We assume throughout the problem that the parameters of the model are such that all consumers buy (one unit of) the good in equilibrium and, therefore, we focus on price pairs such that the market is fully covered.

1.1. (0.5 pt) Find the location of the indifferent consumer as a function of p_A, p_B and t.

1.2. (1 pt) Compute the equilibrium prices and profits.

2. *Investment in "quality"*

Assume now that each firm can invest in increasing the "quality" of its product: firm $i = A, B$ can increase each consumer's gross utility from consuming one unit of its product from $\bar{s}$ to $\bar{s}+\theta_i$ by investing an amount $I(\theta_i) = \frac{1}{2}\lambda\theta_i^2$, where λ is a strictly positive parameter. We consider the following two-stage game:

- Stage 1 (investment): Firms simultaneously choose their "quality" levels θ_A and θ_B.
- Stage 2 (pricing): Firms simultaneously set their prices p_A and p_B.

Assume that λ is such that (i) the second-order conditions of all the relevant maximization programs are satisfied, and (ii) firms choose strictly positive "quality" levels at the (subgame-perfect) equilibrium of the game.

2.1. We solve the game by backward induction. Consider first stage 2.

2.1.a. (0.5 pt) For given θ_A and θ_B, write down the demand functions $D_A(p_A, p_B, \theta_A, \theta_B)$ and $D_B(p_A, p_B, \theta_A, \theta_B)$ of firms A and B respectively (you may restrict attention to prices and "quality" levels such that both demands are strictly positive as this will be the case at the equilibrium of the two-stage game).

2.1.b. (1 pt) Derive the second stage equilibrium prices $p_A^*(\theta_A, \theta_B)$ and $p_B^*(\theta_A, \theta_B)$ for given θ_A and θ_B.

2.2. (2 pt) Consider now stage 1. Determine the (subgame-perfect) equilibrium "quality" levels θ_A^* and θ_B^* chosen by firms A and B. How do θ_A^* and θ_B^* depend on λ? How do firms' equilibrium profits depend on λ? Discuss.

3. *Comparative advertising*

Consider now the baseline model and assume that firms can invest in comparative advertising: firm $i = A, B$ can increase the gross utility derived from the consumption of its product by $\alpha_i/2$ and (simultaneously) decrease the gross utility derived from its competitor's product by $\alpha_i/2$ if it spends an amount $A(\alpha_i) = \frac{1}{2}\mu\alpha_i^2$ in comparative advertising (μ is a strictly positive parameter). Thus, if firm A chooses an advertising intensity α_A and firm B chooses an advertising intensity α_B then the gross utility that a consumer derives from the consumption of firm A's product is $\bar{s} + \frac{1}{2}(\alpha_A - \alpha_B)$ and the gross utility that a consumer derives from the consumption of firm B's product is $\bar{s} + \frac{1}{2}(\alpha_B - \alpha_A)$. We consider the following two-stage game:

- Stage 1 (advertising): Firms simultaneously choose their advertising intensities α_A and α_B.
- Stage 2 (pricing): Firms simultaneously set their prices p_A and p_B .

Assume that μ is such that (i) the second-order conditions of all the relevant maximization programs are satisfied, and (ii) firms choose strictly positive advertising intensities at the (subgame-perfect) equibrium of the game.

3.1. (0.5 pt) For given α_A and α_B, write down the demand functions $D_A(p_A, p_B, \alpha_A, \alpha_B)$ and $D_B(p_A, p_B, \alpha_A, \alpha_B)$ of firms A and B respectively (you may restrict attention to prices and advertising intensities such that both demands are strictly positive).

3.2. (1.5 pt) Determine the (subgame-perfect) equilibrium advertising intensities α_A^* and α_B^* chosen by firms A and B. How do firms' equilibrium profits depend on μ? Discuss.

4. *Agglomeration of consumers*

Consider now the following extension of the baseline model. Suppose that the consumers, whose mass is still assumed to be equal to 1, are uniformly distributed over $\left[\frac{1}{2} - a, \frac{1}{2} + a\right]$ where $a \in \left]0, \frac{1}{2}\right]$. In other words, the density of consumers in a point of $[0, a[$ or $\left]\frac{1}{2} + a, 1\right]$ is zero while their density in a point of the interval $\left[\frac{1}{2} - a, \frac{1}{2} + a\right]$ is $\frac{1}{2a}$. Note that the baseline model corresponds to the special case $a = \frac{1}{2}$.

4.1. (1 pt) Denote $D_A(p_A, p_B, a)$ and $D_B(p_A, p_B, a)$ the demand functions of firms A and B when they charge p_A and p_B respectively. For a given p_B, what are the values of p_A such that $D_A(p_A, p_B, a) > 0$. Focusing on price pairs (p_A, p_B) such that both firms get a strictly positive demand, compute $D_A(p_A, p_B, a)$ and $D_B(p_A, p_B, a)$.

4.2. (2 pt) Derive the equilibrium prices $p_A^*(a)$ and $p_B^*(a)$. What are the equilibrium prices in the limiting case $a \to 0$? Discuss.

Problem 2 (10 points)
Collusion in prices with differentiated products

Two firms produce (imperfectly) substitutable goods *at no cost* and compete in prices. The demand for firm i's product is given by

$$D_i(p_i, p_j) = 1 - \frac{p_i - \sigma p_j}{1 - \sigma},$$

where p_i and p_j denote the prices charged by firms i and j (for $i \neq j \in \{1, 2\}$) and $\sigma \in]0, 1[$ reflects the degree of substituability between the two goods.

1. *One-shot price competition*

1.1. (1 pt) Determine firm i's best response to its rival's price, $p_i = R_i(p_j)$. Are the decision variables (i.e. prices) strategic substitutes or strategic complements?

1.2. (0.5 pt) Show that the equilibrium prices are $p_1^* = p_2^* = p^* = \frac{1-\sigma}{2-\sigma}$ and the equilibrium profits are $\pi_1^* = \pi_2^* = \pi^* = \frac{1-\sigma}{(2-\sigma)^2}$.

2. *Collusion*

We now assume that the above competition game is infinitely repeated, and that both firms have the same discount factor $\delta > 0$.

2.1. We first study the sustainability of *perfect collusion*, i.e. collusion at the price $p_1^c = p_2^c = p^c$ that maximizes the joint profits of firms 1 and 2 (i.e. the industry profits). More specifically, we consider perfect collusion supported by the following trigger strategies: Each firm charges p^c in the first period and continues to do so in the subsequent periods as long as no firm has deviated; if a deviation occurs in a period t, each firm charges p^* in all periods $t' > t$.

2.1.a. (0.5 pt) Compute p^c and the (per-period) profit π^c each firm makes if both firms charge p^c.

2.1.b. (1 pt) If one firm wants to unilaterally deviate, what is the optimal deviation price p^d and the associated profit π^d (made by the deviating firm) in the period in which the deviation occurs?

2.1.c. (2 pt) Show that perfect collusion (using the above-described trigger strategies) is sustainable if and only if $\delta \geq \delta^*(\sigma)$ where $\delta^*(\sigma)$ is a threshold to be determined. Show that $\delta^*(\sigma)$ increases in σ. Interpret this result.

2.2. (2.5 pt) In this question we assume that $0 < \delta < \delta^*(\sigma)$ so that perfect collusion is not sustainable. Show that there exists a price $\hat{p} \in]p^*, p^c[$ such that collusion using the following trigger strategies is sustainable: Each firm charges $\hat{p}$ in the first period and continues to do so in the subsequent periods as long as no firm has deviated; if a deviation occurs in a period t, each firm charges p^* in all periods $t' > t$. [Hint: Fix δ and consider the limit of the sustainability condition when $\hat{p} \to p^*$.]

2.3. We now assume that $\delta \geq \delta^*(\sigma)$ and that there exists an antitrust authority which investigates the industry in every period. If firms collude, the authority will detect the cartel with a probability $\rho > 0$ and will accordingly make each of them pay a fine $F > 0$ (in the period in which detection occurs). If the cartel is detected, also assume that the authority will prevent the firms from colluding in the future: each firm will earn a per-period profit equal to π^* in all periods following the period in which detection occurs. If firms do not collude they cannot be fined (in particular a deviating firm cannot be fined). We focus on perfect collusion. Denote V^c each firm's expected present discounted value of profits at the beginning of the game (i.e. period $t = 0$) if both firms play the trigger strategies described in 2.1 with the additional feature that reversion to the one-shot Nash equilibrium price p^* can also happen because the antitrust authority detects the cartel at a given period and prevents the firms from colluding in future periods. Note that V^c is also the expected present discounted value of profits at a period $t > 0$ such that the cartel was not detected in any period $t' < t$.

2.3.a. (1 pt) Show that

$$V^c = \pi^c + (1-\rho)\,\delta V^c - \rho F + \rho\delta\frac{\pi^*}{1-\delta}$$

2.3.b. (1.5 pt) Show that there exists a threshold $\bar{\delta}(\sigma, F)$ such that perfect collusion is sustainable if and only if $\delta \geq \bar{\delta}(\sigma, F)$.