

MASTER 1 in ECONOMICS
MASTER 1 ECONOMIE ET STATISTIQUE

Game theory / code : M1S22

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→ durée conseillée pour traiter ce sujet : 1 heure

→ **ATTENTION** : le nom de la matière et son code doivent être **IMPERATIVEMENT** recopiés sur la copie d'examen

Two firms $i = 1, 2$ sell an homogenous good on the same market. They compete by choosing quantities, $q_1 \geq 0$ and $q_2 \geq 0$ respectively. The inverse demand function is linear: $P(Q) = A - Q$ with $A > 0$, where $Q = q_1 + q_2$ is the total quantity.

Part 1. Suppose that production costs are zero, and that firm 1 moves first by choosing q_1 . Then firm 2 observes q_1 , and chooses q_2 . We focus on pure-strategy equilibria.

1. Give the strategy set for each firm.
2. Solve for the subgame-perfect Nash equilibria.
3. Can you find a Nash equilibrium which is not a subgame-perfect Nash equilibrium ?

Part 2. Suppose that production costs are zero for firm 2 only. The game is now as follows. Firstly firm 1 learns whether its unit cost is c or c' , with $0 < c < c'$. Then firm 1 moves first by choosing q_1 . Then firm 2 observes q_1 , and chooses q_2 . We focus on pure-strategy equilibria. We assume that $A > c'$.

1. Give the strategy set for each firm.
2. What are the subgames of this game ?
3. Find a Nash equilibrium. Is it a subgame-perfect Nash equilibrium ?

Part 3. Suppose that production costs are zero and that both firms move simultaneously. Each firm can choose between three levels of production only, $q_i \in \{0, 1, 2\}$ for $i = 1, 2$. We assume that $A > 3$.

1. Represent the game under normal form and define its mixed extension.
2. Give the definition of a strictly dominated strategy. Can you find a strictly dominated strategy for this game? What are the implications regarding the quantities that can be produced at a Nash equilibrium ?
3. Explain (informally) the process of iterated elimination of strictly dominated strategies. Give a condition on A under which the strategy 'I produce one unit' is eliminated when using this process. What can you say about the set of Nash equilibria in mixed strategies in this case?

In the following questions firms cannot choose not to produce, meaning each firm can choose between two levels of production, $q_i \in \{1, 2\}$ for $i = 1, 2$.

4. Solve for the Nash-equilibria in pure strategies (the result can depend on A).
5. Suppose that $A = 9/2$. Represent the best reply function of each agent on a graph and deduce the set of Nash equilibria.