UNIVERSITÉ TOULOUSE 1 CAPITOLE

TSE
Toulouse School of Economics
École d'économie de Toulouse

Année Universitaire 2012-2013
SESSION 1

Semestre 2
Master 1

Game Theory
(durée 2h)

Mardi 14 mai 2013 ~ 9h30 – 11h30

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NB:

No documents. No computers. Nothing but paper and pens.

Please provide clear and rigorous answers, in English or in French (not in both!).

The grading scale is indicative. Time is scarce, so allocate your time wisely across exercises.

Exercise 1 (from Montgomery, QJE 1991) (6 points): consider the following dynamic game with four players: two firms A and B, each with one single open position, and two unemployed workers 1 and 2. First, the two firms simultaneously choose a wage under the constraint that it is positive (w_A for firm A, w_B for firm B). Second, after having observed the wages w_A and w_B the two workers choose where to apply. If both apply to the same position, one of them is chosen at random (with probability 1/2) and gets the position; the other worker is left unemployed. If the two workers apply to different positions, each gets the position he applied to.

Payoffs are as follows. An unemployed worker gets zero. The payoff of a hired worker is the wage he earns. A firm gets zero if the position it offers is left unoccupied; it gets $y - w$ otherwise, where y is the value of production ($y > 0$) and w is the wage paid to the worker.

We are looking for the Subgame-Perfect Nash Equilibria (SPNE) of this two-stage game.

1. Suppose that the wages w_A and w_B are given, and consider the second stage of the game. Put it under normal form, and find ALL Nash equilibria, including equilibria with mixed strategies (you have to discuss according to the values of w_A and w_B).

   Note that there always exists a unique symmetric equilibrium.

2. Now assume that in the second stage of the game players coordinate on the unique symmetric equilibrium. Can you find a Subgame Perfect Nash Equilibrium of the whole game in which firms propose the same wage in the first period of the game?

Exercise 2 (5 points): consider the following dynamic game, with $T \geq 1$ periods. There is a single firm. At each period $t = 0, ..., T-1$, a new worker enters the labor market. The worker learns the history of the game, and then decides whether to work or not to work. After having observed this decision, the firm chooses the wage w_t of the worker, under the constraint $w_t \geq 0$. Finally the worker retires. If $t < T - 1$ we move to next period. The game ends if $t = T - 1$.

For the worker at period t, the cost of working is $c > 0$, and the cost of not working is zero. The payoff of the worker is thus his wage w_t, minus this cost.

For the firm, its profit at period t is the exogenous value of production y (assume $y > c$), minus the wage paid. The firm's payoff is the discounted sum of all profits, with a discount factor $\delta \in]0, 1[$.

1. Find the unique Nash equilibrium of the constituant game ($T = 1$).

2. Can you find a subgame perfect Nash equilibrium of the infinite-horizon game ($T = +\infty$) where on the equilibrium path the firm always pays a positive wage, and each worker decides to work ?

3. What would happen if each new worker could not observe what happened in the past (when $T = +\infty$)?

Exercice 3 (from Bénabou-Tirole, RES 2003) (11 points): consider the following dynamic game with two players: a parent (player P), and a child (player C). The child is training for an exam and decides to make an effort (action E) or no effort (action nE). He fails if he does not make an effort and succeeds otherwise. The parent would like the child to succeed, and thus may decide to promise the child a reward in case of success (action R), or no reward (action nR).

Up to now this is a simple story. We now present the key ingredient, namely that the parent knows whether effort is costly or not for the child. In the first part of this exercise, we assume that this information is also known by the child. In the second part, we assume it is not.

So the timing of the game is as follows. At the first stage, Nature draws the effort cost c : $c = 0$ with probability $1/2$, and $c = 1$ otherwise. At the second stage, P observes the value of c, and promises a reward (R) or not (nR). Finally, at the last stage C observes the move by P, and chooses his effort level (E or nE). In the first part of this exercise, C also observes the draw by Nature. In the second part, C does not observe the draw.

Payoffs are as follows. C gets $V \in]0, 1/2[$ in case of success, and 0 in case of failure; also one has to substract the cost of effort c if C exerted an effort; finally the reward is worth $1/2$ to the child, and has to be added if it was promised by P and the child succeeds. The parent gets $W > 1/2$ if the child succeeds, minus $1/2$ if he has to pay the reward.

Finally we focus on pure strategies throughout.

3.1: the complete information game. **(5 points)**

1. Represent the game under extensive form and indicate the different subgames.
2. Put the subgame associated to $c = 0$ under normal form. Find the pure-strategy Nash equilibria for this subgame. Which of them are subgame-perfect ? Same question for the subgame associated to $c = 1$.
3. Give an example of a strategy for player C.
4. Solve for the subgame-perfect Nash equilibria of the whole game.

3.2: the incomplete information game. **(6 points)**

1. Represent the game under extensive form and indicate the different subgames.
2. Define the strategy set of each player.
3. Suppose that player P decides to play nR at each information set where he has the move. What are the best replies of player C? The expected payoff of player P?
4. Suppose that player P decides to play R at each information set where he has the move. What are the best replies of player C? The expected payoff of player P?
5. A teacher suggests that the parent should give a reward to the child only if the cost of the child is $c = 1$, so as to incentivise him when he has difficulties only. Define the best reply of player C in that case and comment on it. Conclude that the described strategy of player P cannot be played in equilibrium.
6. Consider the following strategy for player P: 'I play R if $c = 0$ and nR if $c = 1$'. Explain why this strategy cannot be played in equilibrium.
7. Conclude by exhibiting a Nash Equilibrium of the whole game.