UNIVERSITÉ TOULOUSE 1 CAPITOLE

TSE
Toulouse School of Economics
École d'économie de Toulouse

Année Universitaire 2012-2013
SESSION 1

MASTER 1

ADVANCED CALCULUS
(durée 1h30)

J. P. IBRAHIM

Lundi 13 mai 2013 ~ 14h00 – 15h30

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Exercice 1. Let $a : I \to \mathbb{R}$ be a continuous function and let y be a solution of the following linear differential equation

$$y' = a(t)y \quad (E)$$

a) Show that $(\exists t \in I;\ y(t) = 0 \Longleftrightarrow y(t) = 0,\ \forall t \in I)$.

b) Deduce that $(\exists t \in I;\ y(t) > 0 \Longleftrightarrow y(t) > 0,\ \forall t \in I)$.

Exercice 2. We consider, for $t \in]0, +\infty[$, the following differential equation

$$y'' - \frac{3}{t}y' + \frac{4}{t^2}y = t \quad (E)$$

a) Check that $y_1 : t \to t^2$ is a solution of the homogeneous equation (E_0).

b) Show, using the Lagrange method, that $y_2 : t \to t^2 \ln(t)$ is an independent of y_1 solution of (E_0).

c) Use the Wronskian method to find the explicit form of all solutions of (E).

Exercice 3. Let (S) be the following non-linear system

$$\begin{cases} x' = y \\ y' = 4\sin(x) - 3y \end{cases}$$

a) Find the equilibrium points of (S).

b) Solve the linearized system around $(0,0)$ and $(\pi,0)$ and draw the corresponding trajectories.

c) Are the points $(0,0)$ and $(\pi,0)$ hyperbolic equilibria of (S) ? Justify your answer

d) Does the point $(0,0)$ a stable equilibrium of (S) ? Explain.

e) Does the point $(\pi,0)$ an asymptotically stable equilibrium of (S) ? Explain.

Barème indicatif : **ex 1** : 4 points. **ex 2** : 8 points. **ex 3** : 8 points.