UNIVERSITÉ
TOULOUSE 1
CAPITOLE

MASTER 1 in ECONOMICS
MASTER 1 ECONOMIE ET STATISTIQUE

Probability Theory / code : M1S17

Lundi 24 Juin 2013 ~ amphi MB1

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J. P. DECAMPS

↪ durée conseillée pour traiter ce sujet : 1 heure
↪ ATTENTION : le nom de la matière et son code doivent être IMPERATIVEMENT recopiés sur la copie d'examen
Documents not allowed

1. Let $(X_i)_{i \geq 1}$ be a sequence of random variables identically independently distributed with mean $\mathbb{E}[X_i] = \beta$. Let N be a random variable taking values in $\mathbb{N}$ with $\mathbb{E}[N] = \lambda$. Assume that N and X_i are independent for all $i \in \mathbb{N}$. Let us consider the random variable $S_N = \sum_{i=1}^{N} X_i$

   (a) Compute $\mathbb{E}[S_N | N = n]$.

   (b) Find $\mathbb{E}[S_N]$.

2. Let X be a random variable with normal distribution $N(m, \sigma^2)$ and K a strictly positive number. Compute $\mathbb{E}[e^X \mathbb{1}_{X > K}]$ (express the result in terms of the cumulative distribution function of a standard normal law).

3. Let (X, Y) be a couple of random variables with joint density $f_{X,Y}(x, y) = \lambda^2 e^{-\lambda y}$ if $0 \leq x \leq y$ and $f_{X,Y}(x, y) = 0$ otherwise .

   (a) Compute the marginal densities f_X and f_Y. Are X and Y independent ?

   (b) Find $\mathbb{E}[X|Y]$ and $\mathbb{E}[Y|X]$.

4. Let X and Y two random variables in $L^1(\Omega, \mathcal{F}, \mathbb{P})$

   (a) Give a characterization of $\mathbb{E}[X|Y]$.

   (b) Let us consider two σ-fields $\mathcal{A}$ and $\mathcal{G}$ with $\mathcal{A} \subset \mathcal{G} \subset \mathcal{F}$. Show that $\mathbb{E}[\mathbb{E}[Y|\mathcal{G}]|\mathcal{A}] = \mathbb{E}[Y|\mathcal{A}]$.