MASTER 1 in ECONOMICS
MASTER 1 ECONOMIE ET DROIT
MASTER 1 ECONOMIE ET STATISTIQUE

Macroëconomics / code : M1S12

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↪ durée conseillée pour traiter ce sujet : 1 heure
↪ **ATTENTION** : le nom de la matière et son code doivent être **IMPERATIVEMENT** recopiés sur la copie d'examen

PROBLEM 1: CONSUMPTION-SAVING DECISION

Consider the problem of choosing an infinite sequence of consumption and capital $\{c_t, k_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^{t-1} \log(c_t),$$

with $\beta \in (0,1)$ being an intertemporal discount factor, subject to an initial value k_0 and a transition equation for capital (budget constraint)

$$c_t + k_{t+1} \leq Ak_t^{\alpha}, \tag{bc1}$$

where $A > 0$ and $\alpha \in (0,1)$ are parameters of a production function which uses capital at time t to produce Ak_t^{α} of a good that can be consumed or used as capital next period.

1 – Wrtie the Lagragian of this problem and the first-order conditions. Derive the intertemporal equation for the optimal consumption path. Can you interpret that as contrasting the marginal benefit of consumption today with the marginal benefit of postponing consumption one period?

2 – Find a recursive expression for k_{t+1}/c_t? Iterate that expression forward to obtain c_t as a function of k_t? Which restriction on the parameters is needed in that forward recursion?

PROBLEM 2: TOBIN'S q MODEL

Consider a firm whose real profit (before adjustment costs) is linear in the firm capital $k(t)$:

$$\Pi(t) = a \times k(t).$$

The firm faces convex adjustment costs in the rate of change of its capital $C(\dot{k}(t)$, with $C'(k(t) > 0$, $C''(k(t)) > 0$, $C(0) = C'(0) = 0$. Depreciation is assumed to be zero, such that

$$\dot{k}(t) = I(t)$$

The firms maximizes the present value of its profits net of adjustment costs:

$$V = \int_{t=0}^{\infty} e^{-rt}\Big(ak(t) - I(t) - C(I(t))\Big)dt$$

with $k(0)$ is given.

1 – Write the Hamiltonian of that problem.

2 – Show that the firm's optimal behavior is characterized by the following equation:

$$1 + C'(I(t)) = q(t) \tag{1}$$
$$a = rq(t) - \dot{q}(t) \tag{2}$$
$$\lim_{t\to\infty} e^{-rt} q(t)k(t) = 0 \tag{3}$$

where q is the shadow value of k.